# High School Mathematical Content Standards

## Courses and Transitions

The high school standards specify the mathematics that all students should study in order to be career and college ready. They are organized into **Conceptual Categories**, which are intended to portray a coherent view of high school mathematics. A student's work with any set of standards crosses a number of traditional course boundaries. For example, the Functions Standards would apply to different courses such as Algebra I or Algebra II.

These standards do not mandate the sequence of high school courses. However, the organization of high school courses is a critical component to implementation of the standards. It is a district decision how to design course offerings covering the mathematics standards. Districts can use the traditional approach of Algebra I, Geometry, and Algebra II or implement an integrated approach. There are various high school math pathways to be considered and likely additional model pathways based on these standards will become available as well.

The standards themselves do not dictate curriculum, pedagogy, or delivery of content. In particular, districts may handle the transition to high school in different ways. For example, many students in the U.S. today take Algebra I in the 8th grade, and in some districts and states this is a requirement. By completing grade 7 standards successfully, students have met the prerequisites and are prepared for Algebra I by 8th grade. The standards are designed to permit districts and states to continue existing policies concerning Algebra I in 8th grade.

[College-Ready:] Another major transition is the transition from high school to post-secondary education for college and careers. The evidence concerning college and career readiness shows clearly that the knowledge, skills, and practices important for readiness include a great deal of mathematics prior to the boundary defined by the words in brackets, [College-Ready:], in these standards. Indeed, some of the highest priority content for college and career readiness comes from grades 6-8. This body of material includes powerfully useful proficiencies such as applying ratio reasoning in real-world and mathematical problems, computing fluently with positive and negative fractions and decimals, and solving real-world and mathematical problems involving angle measure, area, surface area, and volume. Because important standards for college and career readiness are distributed across grades and courses, systems for evaluating college and career readiness should reach as far back in the standards as grades 6-8.

## Narrative of Standards - Geometry

An understanding of the attributes and relationships of geometric objects can be applied in diverse contexts—interpreting a schematic drawing, estimating the amount of wood needed to frame a sloping roof, rendering computer graphics, or designing a sewing pattern for the most efficient use of material.

Although there are many types of geometry, school mathematics is devoted primarily to plane Euclidean geometry, studied both synthetically (without coordinates) and analytically (with coordinates). Euclidean geometry is characterized most importantly by the Parallel Postulate, that through a point not on a given line there is exactly one parallel line. (Spherical geometry, in contrast, has no parallel lines.)

During high school, students begin to formalize their geometry experiences from elementary and middle school, using more precise definitions and developing careful proofs. Later in college some students develop Euclidean and other geometries carefully from a small set of axioms.

The concepts of congruence, similarity, and symmetry can be understood from the perspective of geometric transformation. Fundamental are the rigid motions: translations, rotations, reflections, and combinations of these, all of which are here assumed to preserve distance and angles (and therefore shapes generally). Reflections and rotations each explain a particular type of symmetry, and the symmetries of an object offer insight into its attributes—as when the reflective symmetry of an isosceles triangle assures that its base angles are congruent.

In the approach taken here, two geometric figures are defined to be congruent if there is a sequence of rigid motions that carries one onto the other. This is the principle of superposition. For triangles, congruence means the equality of all corresponding pairs of sides and all corresponding pairs of angles. During the middle grades, through experiences drawing triangles from given conditions, students notice ways to specify enough measures in a triangle to ensure that all triangles drawn with those measures are congruent. Once these triangle congruence criteria (ASA, SAS, and SSS) are established using rigid motions, they can be used to prove theorems about triangles, quadrilaterals, and other geometric figures.

Similarity transformations (rigid motions followed by dilations) define similarity in the same way that rigid motions define congruence, thereby formalizing the similarity ideas of "same shape" and "scale factor" developed in the middle grades. These transformations lead to the criterion for triangle similarity that two pairs of corresponding angles are congruent.

The definitions of sine, cosine, and tangent for acute angles are founded on right triangles and similarity, and, with the Pythagorean Theorem, are fundamental in many real-world and theoretical situations. The Pythagorean Theorem is generalized to nonright triangles by the Law of Cosines. Together, the Laws of Sines and Cosines embody the triangle congruence criteria for the cases where three pieces of information suffice to completely solve a triangle. Furthermore, these laws yield two possible solutions in the ambiguous case, illustrating that Side-Side-Angle is not a congruence criterion.

Analytic geometry connects algebra and geometry, resulting in powerful methods of analysis and problem solving. Just as the number line associates numbers with locations in one dimension, a pair of perpendicular axes associates pairs of numbers with locations in two dimensions. This correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof. Geometric transformations of the graphs of equations correspond to algebraic changes in their equations.

Dynamic geometry environments provide students with experimental and modeling tools that allow them to investigate geometric phenomena in much the same way as computer algebra systems allow them to experiment with algebraic phenomena.

**Connections to Equations -** The correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof.

**Modeling Standards** - Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by the words in brackets as [Specific Modeling Standards:].

## Geometry Standards

| **Congruence** | **G - CO** |
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**Experiment with transformations in the plane.**

- G-CO.1. Demonstrates understanding of key geometrical definitions, including angle, circle, perpendicular line, parallel line, line segment, and transformations in Euclidian geometry. Understand undefined notions of point, line, distance along a line, and distance around a circular arc.
- G-CO.2. Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).
- G-CO.3. Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.
- G-CO.4. Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.
- G-CO.5. Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

**Understand congruence in terms of rigid motions.**

- G-CO.6. Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.
- G-CO.7. Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
- G-CO.8. Explain how the criteria for triangle congruence (ASA, SAS, SSS, AAS, and HL) follow from the definition of congruence in terms of rigid motions.

**Prove geometric theorems.**

- G-CO.9. Using methods of proof including direct, indirect, and counter examples to prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.
- G-CO.10. Using methods of proof including direct, indirect, and counter examples to prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.
- G-CO.11. Using methods of proof including direct, indirect, and counter examples to prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.

**Make geometric constructions.**

- G-CO.12. Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.
- G-CO.13. Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

| **Similarity, Right Triangles, and Trigonometry** | **G - SRT** |
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**Understand similarity in terms of similarity transformations.**

- G-SRT.1. Verify experimentally the properties of dilations given by a center and a scale factor:
    - A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
    - The dilation of a line segment is longer or shorter in the ratio given by the scale factor.

- G-SRT.2. Given two figures, use the definition of similarity in terms of transformations to explain whether or not they are similar.
- G-SRT.3. Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

**Prove theorems involving similarity.**

- G-SRT.4. Prove theorems about triangles. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely.*
- G-SRT.5. Apply congruence and similarity properties and prove relationships involving triangles and other geometric figures.

**Define trigonometric ratios and solve problems involving right triangles.**

- G-SRT.6. Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
- G-SRT.7. Explain and use the relationship between the sine and cosine of complementary angles.
- [Specific Modeling Standards:] G-SRT.8. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

**Apply trigonometry to general triangles.**

- [College-Ready:] G-SRT.9. Derive the formula $A = 1/2\ ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
- [College-Ready:] G-SRT.10. Prove the Laws of Sines and Cosines and use them to solve problems.
- [College-Ready:] G-SRT.11. Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

| Circles | G - C |
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**Understand and apply theorems about circles.**

- G-C.1. Prove that all circles are similar.
- G-C.2. Identify and describe relationships among inscribed angles, radii, and chords.
  *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*
- G-C.3. Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.
- [College-Ready:] G-C.4. Construct a tangent line from a point outside a given circle to the circle.

**Find arc lengths and areas of sectors of circles.**

- G-C.5. Use and apply the concepts of arc length and areas of sectors of circles. Determine or derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

| Expressing Geometric Properties with Equations | G - GPE |
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**Translate between the geometric description and the equation for a conic section.**

- G-GPE.1. Determine or derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.
- G-GPE.2. Determine or derive the equation of a parabola given a focus and directrix.
- [College-Ready:] G-GPE.3. Derive the equations of ellipses and hyperbolas given foci and directrices.

**Use coordinates to prove simple geometric theorems algebraically.**

- G-GPE.4. Perform simple coordinate proofs. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).
- G-GPE.5. Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).
- G-GPE.6. Find the point on a directed line segment between two given points that partitions the segment in a given ratio.
- [Specific Modeling Standards:] G-GPE.7. Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.

| Geometric Measurement and Dimension | G - GMD |
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**Explain volume formulas and use them to solve problems.**

- G-GMD.1. Explain how to find the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.
- [College-Ready:] G-GMD.2. Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.
- [Specific Modeling Standards:] G-GMD.3. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. *For example: Solve problems requiring determination of a dimension not given.*

**Visualize relationships between two-dimensional and three-dimensional objects.**

- G-GMD.4. Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

| **Modeling with Geometry** | **G - MG** |
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**Apply geometric concepts in modeling situations.**

- [Specific Modeling Standards:] G-MG.1. Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).
- [Specific Modeling Standards:] G-MG.2. Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).
- [Specific Modeling Standards:] G-MG.3. Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).